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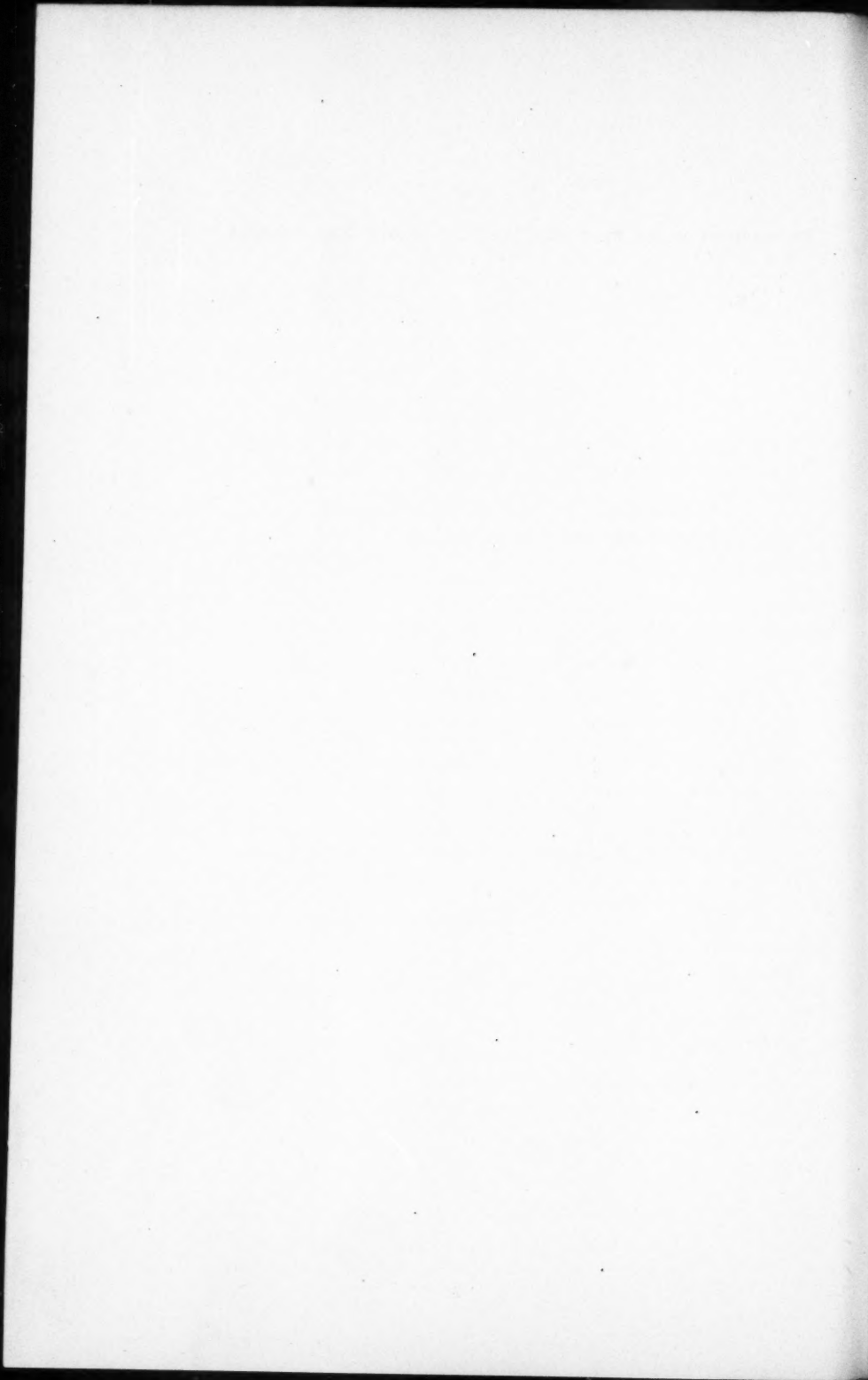
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TYPES OF ABNORMAL COLOR VISION.

BY LOUIS BELL.

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TYPES OF ABNORMAL COLOR VISION.

BY LOUIS BELL.

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DESPITE the fact that so-called color blindness has been studied for more than a century, little is yet known with respect to its real nature or the various forms under which it appears. The present investigation is a preliminary study of the latter made in the hope that it may give at least some slight clues to the former.

It is well known that about one man in 25 has a marked congenital color defect. Without going into detailed statistics, the older data show, from the work of Holmgren, B. Joy Jeffries, and the committee of the London Ophthalmological Society, 3.82 per cent of color deficient in 57398 males examined (Proc. Roy. Soc., LI, 319). This figure is based chiefly on tests with Holmgren's wools, and is materially increased in many later investigations made with the color lantern and the spectroscope. Donders for example found 6.6 per cent of color blind among 2300 railway employees in Holland, and even this figure has been much exceeded. Bearing in mind that minor color defects easily escape the wool test when applied in the ordinary course of testing men for the red and green vision required for signal lights, it is easy to realize that small variations from the normal, although perhaps of much theoretical significance, readily escape notice. These cases, too, have gone undetected on account of the frequent acceptance of Hering's theory, which associates defects in red and in green vision, a position now known in virtue of Burch's work on temporary induced color blindness (Phil. Trans., B 191, 1) to be quite untenable. This work and the earlier experiments of Rayleigh (Nature, 25, 64) have opened up a wide field for the study of abnormalities in color vision, which has of late years been somewhat explored. That such are exceedingly common is in no wise better shown than by a later paper of Burch (Phil. Trans., 199, B, 231) in which beside giving various interesting cases of color defects, the author cites the detailed examination of eleven cases of practically normal color vision.

The red-green and blue-green junctions found for these eleven persons are plotted as points in Figure 1 over the primary color sensation curves, reduced to equal area, as given by Exner (Wien. Sitz. II, 111, 837). The arrows show the mean values adopted by Burch and agree with Exner's junctions to a close approximation. The extreme points at both junctions show a degree of variation in the sensation curves which connotes slight abnormality easily detectable with a spectroscopic test. The average red-green junction was at w. l. 5842 and the blue green junction at w. l. 4946. In Burch's earlier paper (*loc. cit.*) in the examination of 70 persons, all of whom passed the Holmgren wool test, still greater variations in the junction points appeared. And these differences merge again into those shown by patients recog-

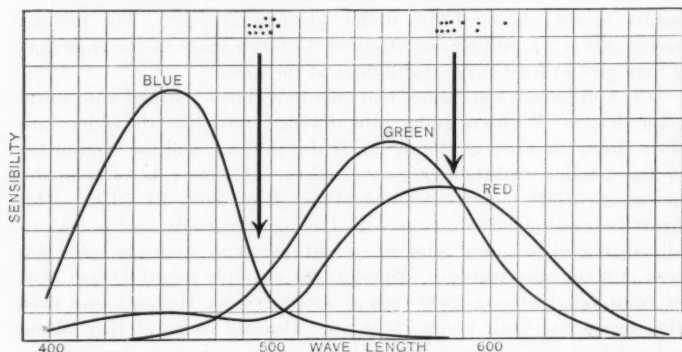


FIGURE 1.

nized as color-blind by simple tests, so that while one may define an average normal color-vision, from which large variations are rare, it is clear that the relations of the color sensations are subject to considerable variability. It is the purpose of this paper to classify such variations with reference to their bearing on the general color sense of the individual and to point out such of the more conspicuous abnormalities as have been detected in tests of color-sense.

The facts will be stated in terms of the ordinary trichromatic theory for the sake of simplicity. How far the observed facts regarding the typical color-sensation curves may be complicated by aberrancies of color perception referable to the cortex alone, and to what degree the independent violet sensation postulated by Burch (*loc. cit.*) may be

involved with the residuum of the red and green sensations, the yellowing of the lens, and variations of macular pigmentation, are matters to be considered when the simpler facts of sensation can be better co-ordinated.

The relations of the three fundamental color sensations of the Young-Helmholtz theory, at least rest on a sound experimental basis and it is well understood that ordinary cases of color blindness imply an easily measurable deficit in one of these sensations.

In the normal or average eye the three fundamental sensations are related in a certain normal manner which may be schematically represented as

$$R, G, B,$$

in which each of the sensations has its typical average value. Similarly the ordinary case of red blindness may be written — R, G, B indicating a deficit from the normal value of the red sensation. To be rigorous one should give R a coefficient indicating the relative deficit of the red vision from its normal which may be anything from an amount just recognizable as a variant from normal to 100%. For example a certain case of partial red blindness examined by the writer would have been expressed

$$-.64R, G, B.$$

And similarly one may set down as the variants from normal involving change in a single color sensation curve the following

$$\begin{array}{l} R, G, B, \\ (1) +R, G, B, \\ (2) R, +G, B, \\ (3) R, G, +B, \\ (4) -R, G, B, \\ (5) R, -G, B, \\ (6) R, G, -B, \end{array}$$

Of these (4), the —R type, is the ordinary case of "color blindness"; (5) is the rather rare green blindness of which typical cases are reported by Abney (Proc. Roy. Soc., Vol. 83A); (6) is the still rarer blue blindness (Abney, *Colour Vision*, p. 73). Of the plus variants (1) is the plus red, the first of the type detected, (Rayleigh, loc. cit.). Burch (*Physiological Optics*, p. 119) cites several instances of (2). The plus blue variation is apparently very uncommon but Burch cites a case

(Phil. Trans., 199, B, p. 250) which appears to be a good example. There seems to be no intrinsic reason why plus variations from the normal should be less frequent than the minus variations, and they probably are not so in fact, the apparent reason being the wrong diagnosis likely to be made with the ordinary method of testing. Thus (1) and (5) are quite certain to be confused in the worsted tests most commonly employed, likewise (2) and (4) while plus or minus variations in the blue unless very marked would almost certainly escape detection.

The spectroscope and the color mixing apparatus give the only reliable methods of testing for minor variations, and the writer has used in his own experiments a simple form of spectroscopic test which may be worth describing here. The chief test is in principle the converse of Rayleigh's, and consists in matching a synthetic yellow and a synthetic blue-green, of the same hues as the spectral colors corresponding to the red-green and blue-green junctions for the normal eye, by shifting a pure spectrum which occupies the lower half of a slit in the focal plane of the eyepiece while the synthetic color occupies the upper half.

The apparatus is shown in diagram in Figure 2. The basis is a simple constant deviation spectroscope mounted in a capacious wooden box to avoid stray light. The prism P is mounted as usual on a turntable b , rotated by the milled head D with a screw bearing on a steel plane on the turntable. On the screw shaft is a wide pinion f , engaging a narrow rack ending in a pointer g , moving over the scale h . The rack works under the turntable freely through guides. The main slit, S , has the usual collimating lens c , of 4 cm. diameter and 15 cm. focus. The observing telescope O , of about 4 cm. diameter and 35 cm. focus has a compound eyepiece slit e , a slider with an adjustable slit and a clear aperture for viewing the spectrum. Long screws k, k , position the slider so that when slipped to the right the slit can be brought anywhere in the spectrum, and when slipped to the left the field is wholly or partly clear.

Beyond the turntable is a second collimator with adjustable slit S' and lens c' 12 mm. in diameter. This ranges over the prism P which is 25 mm. thick and the beam from c' is turned into the upper part of O by the mirror m . A small 60 degree prism can also be placed in front of c' to furnish a reference spectrum. Condensing lenses L, L' direct the light from sources l, l' upon the slits.

In using the apparatus the filter i is placed in front of S' . For synthetic yellow this is made of opposed wedges of cobalt and selenium

glass, which by adjustment of S' , L' and l' can be made to give a pretty good match to the normal eye with pure spectral yellow, brought to the same luminosity by the main slit. The patient is then, the spectrum having been widely displaced by turning D , required to bring it back for a match. A very slight degree of red blindness causes him to match the synthetic yellow with a green, while a plus red or minus

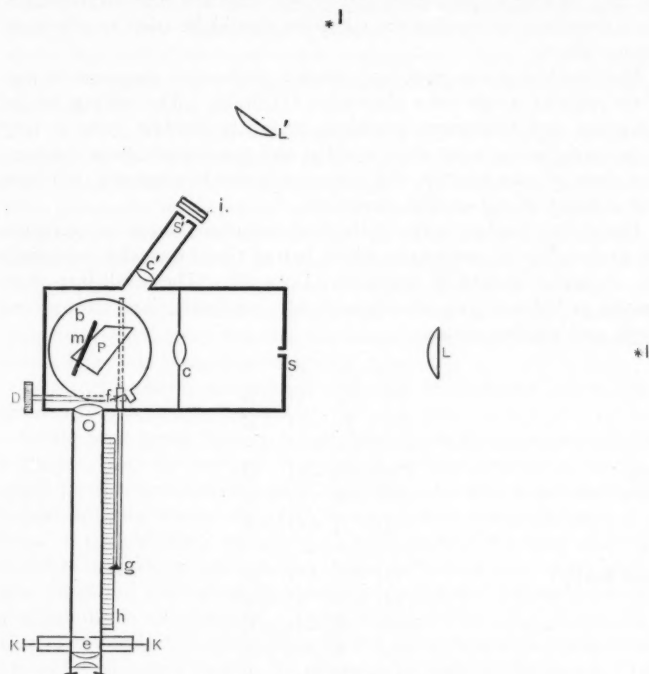


FIGURE 2.

green color-abnormality will produce a reddish match. Filter i is then replaced by a synthetic blue-green, produced by cobalt chloride in acetone solution combined with a uranine filter, and the test repeated. Here a green blind observer would match with blue and a blue blind case with green. This blue-green junction supplements the other in the diagnosis. Finally for further evidence of $+$ or $-$ red

and blue sensations, the red and violet end points are determined after resting the eye. This requires care in keeping the slit width and illumination constant and in resting the eye for ten minutes or so before the test to approach a steady condition of adaptation. The two junction point filters should be modified by neutral tint glass if needed, so that one need not adjust their luminosity with the slits, but may be able to pass quickly from one standard filter to the other. Both directions of moving the spectrum should be tried to eliminate fatigue effects.

This method gives quick and certain qualitative diagnosis of any of the cases of simple color aberration (1) to (6). The settings at the red-green and blue-green junctions instantly disclose even a very slight variation in color sense and the end point readings in conjunction show at once whether this variation is due to weakness or abnormal strength of red or blue sensations.

Proceeding further in the analysis of variations one must recognize the probability of variants in which two of the three color sensations are abnormal instead of one as in (1) to (6). These fall into three groups as follows: two sensations weak; two sensations strong; one weak and another strong.

(7) $-R, -G, B$

(8) $-R, G, -B$

(9) $R, -G, -B$

Also

(10) $+R, +G, B$

(11) $+R, G, +B$

(12) $R, +G, +B$

and finally

(13) $-R, +G, B$

(14) $-R, G, +B$

(15) $R, -G, +B$

(16) $R, +G, -B$

(17) $+R, G, -B$

(18) $+R, -G, B$

These twelve variations are less easy of diagnosis than (1) to (6) since they depend on more complex quantitative relations.

Take for example (7). Here the red-green junction may be absolutely normal, but the blue-green junction will yield a bluish match showing either weak green or strong blue. The red end point, or for that matter the blue end point, tells the story. Case (8) can be dis-

tinguished from case (2) by the end points, and so on, assuming that the abnormalities are not vanishingly small.

The red end point gives fairly definite information but the blue end point is less satisfactory. It is so much modified at times by variation in pigmentation of the macula and by yellowing of the lens as to be somewhat confusing. In the old the lens may be so altered in color that the solar H and K lines cannot be seen. As to pigmentation a case is cited by Abney (*Researches in Colour Vision*, p. 349) in which much of the blue end of the spectrum was absorbed. Such cases are separated from a genuine -B case by their giving, with R and G normal, a normal blue-green junction.

It is pertinent to inquire in how far these twelve binary variations actually occur. The ordinary cases of color blindness reported have not been so tested as to show them easily. The writer has never noted them personally in congenital color blindness, but one (7) is typical of color fatigue due to the mercury-arc and may easily be detected, as found by Williams and the writer (*Electrical World*, Sept. 2, 1911). A case reported by Edredge-Green (*Colour Blindness*, p. 154, F. A.) probably belongs to this type, since with a considerably shortened red spectrum his red-green junction was toward the green and he classified violet, blue, and bluegreen together.

Type (8) probably corresponds to a case described by Burch (*Phil. Trans.*, 199, B, p. 250, XVII).

Of the next group there is much difficulty in obtaining definite information since for example R,+G,+B and the ordinary -R, G, B could be distinguished only with some difficulty and would certainly escape ordinary tests. Any case in which some of the ordinary mistakes of the red blind are made with the confusion colors, while the ordinary end points are retained should be looked into carefully. A plus sensation will rarely be detected by the end points unless the abnormality is very marked. Burch's fatigue tests (*Phil. Trans.*, 191, B, p. 1 et seq.) used to supplement the junction and end point tests probably will prove to give the clearest diagnosis of this group. Luminosity tests might be useful since there is evident increase in general sensibility but on account of difficulties due to adaptation and the uncertainty of readings, the experiments are troublesome unless with experienced observers.

The next group (13) to (18) is easier to deal with and more examples may be found in the literature. (13) is well shown by Burch (*Phil. Trans.*, 199, B, p. 240, VII) in a case of marked but not complete red-blindness with slightly hypernormal green. Type (14) is also clearly

described in Burch's very next case (*loc. cit.* p. 241, VIII), and (15) is also described in the same paper as case XII. To (16) should probably be referred a case described by Edridge-Green (*Colour Blindness*, p. 140, C. A.) in which the red end point was normal, and the blue considerably shortened while the ordinary yellow was encroached upon by the green and the blue-green junction shifted towards the blue. The remaining two types of this group do not appear in any recorded cases examined by the writer. (17) and (9) might easily be confused, as also (18), with some degrees of ordinary green blindness.

Finally there must be recognized a group of abnormalities in which all three primary sensations are affected. In the notation here used one may have $+R, +G, +B$ and $-R, -G, -B$, as well as the normal R, G, B . In other words some persons undoubtedly have a generally strong color sense, and others a generally weak color sense, in each case without peculiarities. From each of these types obviously may spring a group of color variants with a single abnormality, corresponding to the (1) to (6). In these there is simple variation of one sensation with a general sensibility graded up or down.

Likewise there will be groups corresponding to variations, $+$ or $-$, of two color sensations in the same direction, giving simple binary variations graded up or down from (7, 8, 9) and (10, 11, 12).

If one sensation remain $+$ or $-$, with the other two abnormal relatively in opposite directions, there results a group like (13-18) but starting from a different plane of sensibility; and, since the $+$ and $-$ are referred to the normal as the datum point, the types are sometimes sharply marked. This group is, in effect made up of ternary color aberrations in which all three primary sensations show abnormal values. It comprises the following types.

- | | |
|--------------------|--------------------|
| (19) $+R, +G, +B,$ | (23) $-R, -G, -B,$ |
| (20) $+R, +G, -B,$ | (24) $-R, +G, -B,$ |
| (21) $+R, -G, +B,$ | (25) $-R, -G, +B,$ |
| (22) $+R, -G, -B,$ | (26) $-R, +G, +B,$ |

Of this ternary group several of the types are to be found more or less clearly described in the literature. (21) for example, is substantially Burch's Case XIII, (*Phil. Trans.*, 199, B, *cit.*) where red and blue sensations were abnormally strong with marked deficiency in green. A rather clear case of (23) is described by Edridge-Green (*Colour Vision*, p. 158, G. A.) Here the spectrum was clearly shortened at both ends, especially the red, while some of the color matches indicated weak green sensation as well. The patient evidently had a general

low degree of color sense with particular weakness of red sensation. (24) corresponds with a description by Burch (*Physiological Optics*, p. 119, of a case of predominant green sensation so marked that vision was almost monochromatic, which makes it probable that the red and blue sensations were weakened. Indeed it seems likely that great exaggeration of one sensation may be associated with weakness of one or both the remaining sensations. (25) seems to agree well with a case cited by Edridge-Green (*Colour Blindness*, p. 203) as examined by Sir Wm. Ramsay. In this case there was remarkable exaggeration of the blue, and degradation of the other sensations, so that vision was almost monochromatic.

All the color abnormalities here noted are such as belong to the simple trichromatic theory assuming that the several color sensation curves retain their shapes and their normal position in the spectrum, varying only in area. So little is known of the mechanism of color vision that one cannot even predicate whether shifts and changes of shape are or are not likely to take place. Indeed these could hardly be differentiated from other variations except they chanced to be very marked indeed. Even the interesting case described by Abney and Watson (*Proc. Roy. Soc.*, 89, A, p. 232) as showing a shift of the green sensation toward the red should be tested by the junction and end points and by Burch's fatigue method before forming a final judgment.

If this shifting, or a change of shape in the sensation curve should prove real, still further classes of variants would be formed, but however that may be, the mere variations of sensation curve area which are known to take place must give rise, assuming a certain relation between them as normal, to the definite groups of color variants here noted.

Of the 26 abnormal types of congenital color vision in this list 16 are fairly represented in recorded cases. Seven of the remaining 10 are types having two sensations + and thus varying from a simple deficit of the remaining sensation only in the degree of luminosity of the other two. The best method of differentiation here seems to be careful study of the end points, and fatigue tests. The remaining three involve abnormal blue vision combined with abnormal green vision, both of which separately seem to be relatively rare. Whether they are so in fact is somewhat dubious since with the commoner tests all the smaller variations in the blue are likely to be missed or merged in variations of pigmentation or color in the lens, while certainly most of the + G types are placed with the more familiar -R. In all the

study of minor variation in color sensations the color fields ought to be more thoroughly investigated than is usual.

In conclusion the following list shows the tabulation of all types of abnormality here considered, those which the writer has observed or found recorded being denoted by an asterisk.

<i>Simple</i>	<i>Binary</i>	<i>Ternary</i>
(1) +R, G, B*	(7) -R, -G, B*	(19) +R, +G, +B
(2) R, +G, B*	(8) -R, G, -B*	(20) +R, +G, -B
(3) R, G, +B*	(9) R, -G, B	(21) +R, -G, +B*
(4) -R, G, B*	(10) +R, +G, B	(22) +R, -G, -B
(5) R, -G, B*	(11) +R, G, +B	(23) -R, -G, -B*
(6) R, G, -B*	(12) R, +G, +B	(24) -R, +G, -B*
	(13) -R, +G, B*	(25) -R, -G, +B*
	(14) -R, G, +B*	(26) -R, +G, +B
	(15) R, -G, +B*	
	(16) R, +G, -B	
	(17) +R, G, -B	
	(18) +R, -G, B	

The writer will be grateful for notes on any of the missing types in the table which may have escaped notice in the very scattered literature of this intricate subject.

A study of these types inevitably leads to the question as to whether any remedial measures can help the victims of abnormal color vision. Within restricted limits the answer may be affirmative. The method which has to be followed is precisely that which has already been tried with considerable success in modifying the color of artificial illuminants to obtain normal daylight values of color viewed by them. An ordinary gas flame, for example, is in effect partially blue-blind and the normal eye will see colored objects under such a light very much as the partially blue blind would see them in daylight. The necessary correction has been found to be the interposition of absorbing media which reduce the green and red elements in the same degree as the deficit of the blue element in the source. The penalty of doing this is the loss of considerable luminosity. The selective screens for this purpose are highly effective subject to this limitation. By a process exactly analogous it should be possible to provide a certain proportion of the partially red blind with spectacles which would give them at least an approximation to normal color vision, although at the expense of con-

siderable luminosity, in amount depending on the extent of the red sensation deficit. Those in whom one sensation is nearly or quite absent are of course beyond the chance of help, since balance would require an almost complete obscuration of the remaining sensations, but the theory of the correction rests on a substantial basis and can be put into practice in not too severe cases of partial color deficiency. The method to be followed would be substantially that of Abney in obtaining the sensation curves of the individual and the problem then would resolve itself into making approximate corrections to reduce the curves as nearly as possible to normal relations. In bright light the corrected vision would then show colors in approximately their true relations, though somewhat dulled.



